# Advanced Algorithm

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Lecture 8: Combinatory Algorithm

# Knapsack Problem

- Ref: Approximation Algorithm, Chapter 8
- Given a set $S = \{a_1, \cdots, a_n\}$ of objects, with specified size and profits (size($a_i$) and profit($a_i$)), and a knapsack capacity $B$. Find a subset of objects whose total size is bounded by $B$ and total profit is maximized.
- Algorithm 1: greedy algorithm
  - Analysis: arbitrary bad (homework)
  - Improvement (homework)
- Algorithm 2: a pseudo-polynomial time algorithm (dynamic programming)
- Algorithm 3: rounding technique
  - Analysis: $(1-\epsilon)$-approximation ratio for any $\epsilon > 0$

# PTAS

- PTAS: polynomial time approximation scheme (For example: $n^{1/\epsilon}$)
- FPTAS: fully polynomial time approximation scheme (For example: $poly(n, 1/\epsilon)$)
- APX: constant-factor approximation algorithms
- FPTAS $\subseteq$ PTAS$\subseteq$ APX

**Theorem**

*Unless $P = NP$, $FPTAS \subsetneq PTAS \subsetneq APX$*

# Steiner Tree

- Ref: Approximation Algorithm - Chapter 3
- Giver an undirected graph $G = (V, E)$ with non-negative edge costs, where vertices are partitioned into two sets: Required set $R$ and Steiner set $S$, find a minimum cost tree in $G$ which contains all the required vertices and subset of Steiner vertices.
- Restricted version: metric Steiner tree problem (weight function satisfies the triangle inequality)
- There is an approximation factor preserving reduction from the Steiner tree problem to the metric Steiner tree problem.

# Metric Steiner Tree

- Algorithm: based on minimum spanning tree
- Analysis: 2-approximation ratio, tight
- Current results
  - 1.55-approximation ratio, (2000)
  - 1.39-approximation ratio, (2010 STOC)
  - APX-complete (cannot be approximated within $\frac{96}{95}$) (2008)

# Travelling Salesman Problem

- Giver a complete graph with non-negative edge costs, find a minimum cost cycle visiting every vertex exactly once.
- Hardness of approximation: cannot be approximated within a factor of $\alpha(n)$ for arbitrary polynomial time computable function $\alpha(n)$, unless $\mathrm{P} = \mathrm{NP}$.
- metric TSP
- Algorithm: based on minimum spanning tree
  - Analysis: 2-approximation ratio, tight
  - Improvement: 1.5-approximation ratio, tight
  - currently best result
- APX-complete

# Homework

- Approximation Algorithn - Problem 8.1, page 72 (Knapsack)
- In the knapsack problem, order all items according to profit/size. Suppose the algorithm in Question 1 selects $k$ items. Modify the algorithm as max$(\mathrm{profit}_1 + \cdots + \mathrm{profit}_k, \mathrm{profit}_{k+1})$. Show the approximation ratio of this modified algorithm is 1/2.
- Approximation Algorithm - Problem 3.2, page 34 (Steiner Tree)